



# Current Algebra PoW

The Math Forum's PoWs provide non-routine constructed response problems. The Algebra problems target concepts typically learned in an Algebra I class. Memberships and mentoring options are available at the individual, class, school, and district levels.

## Alvin's Theorem – posted March 24, 2007

Ms. Powers asked her class to look at a list of square numbers and see if they could find any interesting patterns. After a few minutes, Alvin raised his hand and said, "Ms. Powers, I think I've found something cool. If I take two consecutive squares and subtract them, the difference is always the sum of two consecutive integers."

"Show the class what you mean by that, Alvin," said the teacher.

Alvin wrote the following on the board:

$$\begin{array}{l} 49 - 36 = 13 \text{ and} \\ 13 = 6 + 7 \\ \\ 64 - 49 = 15 \text{ and} \\ 15 = 7 + 8 \end{array}$$

Turning to the class, he said, "I call this 'Alvin's Theorem.'"

Ms. Powers smiled and said, "Very good, but if you want to call it a theorem, you must be able to prove that it's true for every possible pair of consecutive square numbers. Can you do that using algebra?"

Alvin worked for a while and then said, "Yes, I can do that, too. Here's how."

What might Alvin have written on the board next?

**Extra:** Later, Alvin found a similar pattern for the difference of every other square, such as  $49 - 25$  or  $64 - 36$ . What pattern involving a sum do you think he discovered this time?